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I also demonstrate the property above referred to, viz: That after one-half of the digits are found the second half may be obtained by subtracting each figure already obtained successively from 9, as follows:

Let $\frac{1}{d}$ be a fraction which when reduced to a decimal gives a repetend in which the remainder $d-1$ occurs We evidently have

$$\frac{1}{d} = .R\,\frac{d-1}{d}, \text{ and } \frac{d-1}{d} = .R'\,\frac{1}{d};$$

in which R contains the digits before reaching the remainder $d-1$, and R' the digits after reaching that remainder. But

$$\frac{1}{d} + \frac{d-1}{d} = 1;$$

hence the sum of the second members of the equation must also equal 1. R' must contain the same number of places as R, since it contains the same sequence of figures differently arranged. Now since the fractional parts when added give 1, $R + R' = 99\,.\,.\,.\,.\,.\,9$ repeated as many times as there are digits in $R + R'$, whence $R = 99\,.\,.\,.\,9 - R'$. Q. E. D.

The *second principle* may be proved as follows: As has been shown, the remainder next to the last is

$$r_2 = \frac{d\,q_1 + 1}{10}.$$

In order that this may be an integer the *units* of $d\,q_1$ must be 9; and it is evident that if the units of d be 1, then q_1 must be 9; if it be 3 then q_1 must be 3; if it be 7 then q_1 must be 7; but if it be 9 then q_1 must be 1.

In conclusion I will remark that it is improper to call repetends, as they are usually represented, *decimals*. For an example. $.\dot{3}$ is not a decimal, but should be read *three-ninths;* so $.\dot{1}\dot{6}$ should be read *sixteen ninety-ninths;* $1\dot{4}$ should be read $1\frac{4}{9}$ *tenths*, &c.

SOLUTION OF A PROBLEM.

BY G. W. HILL, ESQ., NYACK TURNPIKE, N. Y.

The following problem appeared in the *Mathematical Monthly*, (Vol. 1, p. 29), and no solution was published in that periodical:

"Show that the product of six entire consecutive numbers cannot be the square of a commensurable number."

Since the square root of every integer, not an exact square, is a surd, it will be sufficient to show that the product cannot be the square of an integer. Let the six numbers be denoted, n being an odd integer, by

$$\frac{n-5}{2},\ \frac{n-3}{2},\ \frac{n-1}{2},\ \frac{n+1}{2},\ \frac{n+3}{2},\ \frac{n+5}{2}.$$

Then it is required to prove the impossibility of

$$\frac{n^2-25}{4} \cdot \frac{n^2-9}{4} \cdot \frac{n^2-1}{4} = \square.$$

Let us put

$$\frac{n^2-9}{4} = x,$$

where x is integral since it is the product of two integers. Then it will suffice to prove the impossibility of

$$x(x+2)(x-4) = \square.$$

Let us suppose $x = k^2 y$, where k^2 is the largest square factor contained in x, and thus y will be divisible by no square other than unity. Then we have to prove the impossibility of

$$y(k^2 y + 2)(k^2 y - 4) = \square.$$

But since y contains no square factor, both members of this equation must be divisible by y^2; this demands that $2 \times 4 = 8$ be divisible by y. Hence, having regard to the restriction on the form of y, if the equation is possible, it can be so only for the values $y = 1$ or $y = 2$. The first gives

$$(k^2-1)^2 - \square = 9,$$

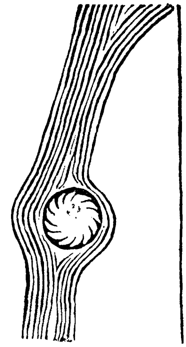
which is satisfied only by $k^2 - 1 = 5$, or $k = \sqrt{6}$, a surd, thefore y cannot be unity. For $y = 2$, we have

$$2(k^2+1)(k^2-2) = \square.$$

But every square is of the form $3n$ or $3n + 1$; if these are substituted in succession for k^2 in the left-hand member of the last equation, it will be seen that the resulting quantities are of the form $3n + 2$, and thus cannot be squares. Therefore y cannot be 2, and the impossibility is completely demonstrated.

Evidently the proposition might be enunciated in the much more general manner:—

The product of any number of consecutive integers cannot be an exact power of any degree.



Prof. J. E. Oliver, Cornell University, Ithaca, N. Y. writes:

Your correspondent's "query" is answered by some standard writer on Physics (probably Dagain or Ganot, or both,)—and the picture there is better than I can draw.

The stream, adhering to the ball, is deflected toward it. If the ball is to the right of the stream's axis more water passes to the ball's left and is deflected to the right than the reverse; $\therefore$ the stream is, on the whole, deflected to the right, $\therefore$ the ball is drawn to the left, since action and reaction are opposite.

* * *

Note on the Ellipse.—From the well known property of the ellipse, that the rectangle of the abscissas is to the square of the ordinate as the square of the major axis is to the square of the minor axis, we get the equation

$$y = \frac{b}{a} \sqrt{a^2 - x^2},$$

in which a and b are the semi-axes and x and y the co-ordinates; the origin being at the center. If we put

$$x = \frac{2\,a\,n}{n^2 + 1} \text{ or } \frac{a(n^2 - 1)}{n^2 + 1},$$

y will always be rational and equal to

$$b \cdot \frac{n^2 - 1}{n^2 + 1} \text{ or } b \cdot \frac{2\,n}{n^2 + 1}.$$

Assume $n^2 + 1 = a$, then will $2n$ and $n^2 - 1$ represent the abscissas corresponding to the ordinates

$$b \cdot \frac{n^2 - 1}{n^2 + 1} \text{ and } b \cdot \frac{2\,n}{n^2 + 1},$$

where n may be any number whatever greater than *one*.

Hence, if we divide the semi-major axis of any ellipse into $n^2 + 1$ equal parts we may find two points in the axis whose distance from the center or from the extremity of the axis may be expressed in integral parts of n, and each of which has a rational ordinate.

Example.—Put $n = 2$, then is the semi-major axis $= n^2 + 1 = 5$, and the two points in the axis are respectively $2\,n$ and $n^2 - 1$, or 4 and 3 dis-